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Towards Personalized Mathematics Learning: Developing an Instrument to Map Students' Mathematical Content Knowledge (MCK) about Function as a Foundation for Permutation Group Concepts

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Abstract

Mathematics learning at the college level requires an in-depth knowledge of the prerequisite concepts that have been previously studied. Function is a fundamental concept that serves as an introduction to understanding the permutation group. This study describes the development of an instrument that measures the students' mathematical content knowledge (MCK) level about function. The processes of development consist of four steps that include: identifying aspects of knowledge about function, developing the questions, checking the content validity, and improving the quality of questions. Four aspects were synthesized that include: basic function knowledge, operation in function, composite function, and properties of function. Ten questions were arranged as the initial product then were validated by two experts in terms of the content. Based on their suggestions, spelling errors were corrected, question sentences were simplified, the number of questions was decreased, the information about the curve was added, and to guarantee that the developed instrument measured what it should be measuring. The final instrument consisted of six questions. The readability test was conducted by the students to see the practicality of the instrument. The result of this study shows that the developed instrument can be used to measure the level of students' MCK about function.

Keywords: Function; Mathematical Content Knowledge; Personalized Mathematics Learning; Permutation Group

Introduction

The research reported here is part of a bigger project, design research for facilitating students to do mathematical abstraction of the permutation group concept. To increase the transferability of that design research, the researchers focus on the research and design instrument that emphasizes the characteristics of the design research subject. Hence, the researchers developed an instrument to capture the level of mathematics education students' mathematical content knowledge about function. The function topic is chosen because this topic is a prerequisite for students before they learn about the permutation group.

Mathematics learning at the college level requires an in-depth knowledge of the prerequisite concepts that have been previously studied (Rach & Ufer, 2020). These concepts do not stand alone, but are interconnected in a complex mathematical framework. One topic that mathematics education students study in abstract algebra courses is permutation groups. Unfortunately, many students have minimal knowledge regarding this topic because they focus more on memorizing formulas and procedures, rather than on understanding the meaning and relationships between mathematical concepts (Idris, 2009). Function is a fundamental concept that serves as an introduction to understanding the permutation group. When students' knowledge about function is merely procedural, students tend to be conceptually unprepared to access more complex abstract ideas (Arslan, 2010).

The uniqueness of teaching mathematics is currently receiving a lot of attention. It is often believed that teachers must possess precise pedagogical and subject matter content knowledge in order to teach the subject matter effectively (Ball et al., 2008). Subject matter content knowledge, pedagogical content knowledge, and curriculum knowledge are the three domains of knowledge that Shulman (1986) initially proposed as being necessary for teaching. Subject matter content knowledge could be considered Mathematics Content Knowledge (MCK) in the context of teaching mathematics. In this context, the idea of MCK becomes highly relevant. MCK refers to fundamental mathematical definitions, concepts, methods, and procedures (Ekawati et al., 2015).

For students of the mathematics education department, MCK is fundamental in enabling them to not only perform mathematical tasks but also to explain concepts clearly, recognize student misconceptions, and present mathematical ideas in various representations. This perspective highlights the importance of assessing preservice teachers' MCK—particularly in foundational topics such as function—before engaging with more abstract topics like permutation groups. Developing instruments that can effectively show students' MCK will therefore support more personalized and targeted learning interventions.

Assesments are used in the educational context to monitor and ascertain students' performance as well as to give diagnostic information (Zhang et al., 2022). Unfortunately, overall psychological measurements have typically been used to rank individual students, which does not meet the growing demand for personalized evaluations. The instruments currently available are generally common and have not been specifically designed to assess the students' MCK to understand algebraic structures such as the permutation group. In fact, the personalized learning approach demands accurate and information-rich diagnostic data about the extent of students' conceptual readiness.

Referring to the theoretical framework adapted by Karahasan (2010), student's MCK can be classified into three levels that include level 0, level 1, and level 2. Level 0 is characterized by not expressing the definition correctly, not using appropriate notation sensibly, only answering questions

that require declarative knowledge, not using different representations, and having difficulty when there is a need to see connections between different topics. Level 1 is shown by expressing the definition correctly, using appropriate notation sensibly, only answering questions that require declarative and/or procedural knowledge, using different representations, and seeing the connection between different topics. Meanwhile, level 2 is stated by expressing the definition correctly, using appropriate notation sensibly, answering questions that require all types of knowledge (declarative, procedural, and conditional), using different representations easily, and seeing the connection between different topics and moving among them smoothly.

By understanding this level of classification, developing this kind of instrument is crucial for educators to accurately identify each student's MCK position. This research aims to develop an instrument capable of mapping students' MCK about functions based on these levels, so that the results can be used as a basis for developing more targeted, adaptive, and personalized learning strategies learning strategies for permutation group topics.

Research Methodology

In the process of development, four phases adapted from Abdullah & Halim (2010) were conducted. The steps were: (1) Step 1: Identifying aspects of knowledge about function. The researchers looked for the constructions that made up the idea of function in the literature. From the obtained literature, the researchers derived the constructs to represent the scope of knowledge about function required to assess students' readiness before studying the permutation group; (2) Step 2: Developing questions. For every construct, questions were created. There were ten questions created in total at first. However, due to the short work duration, this number was reduced to six questions. (3) Step 3: Checking questions validity. Two algebra experts evaluated the instrument's face and content validity. (4) Step 4: Improving the questions. Questions would be improved based on advice from the experts for better quality of questions.

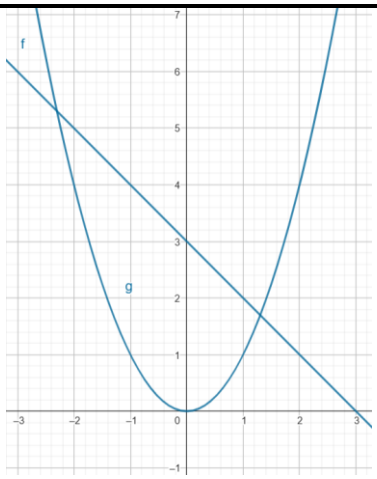
Findings and Discussion

This section describes the process of developing an instrument to assess the students' MCK about function. In terms of aspect identification, the researchers divided it into four aspects that include (1) basic function knowledge, (2) operations on function, (3) composite function, and (4) properties of function. Meanwhile, the researchers arranged the questions into three different knowledge types that include declarative, procedural, and conditional knowledge. This is in line with Mason & Spence (1999) that obtaining knowledge needs having three iterative phases: (1) declarative knowledge forms the base and oncoming actions (knowing that), (2) conditional knowledge gives a sense of direction and supplies a summary for the actions (knowing why), and (3) procedural

knowledge produces actions and applications of knowledge (knowing how). In terms of the question development, the researchers built upon the consideration of MCK level characteristics by Karahasan (2010). It was obtained ten questions which consist of different aspects and knowledge types as presented in Table 1.

Table 1: Initial Questions

No	Aspect	Knowledge Type	Origin	Objective(s)	Question(s)
1	Basic function knowledge	Declarative	Adopted from Karahasan (2010)	Define the concept of function.	What do you think a function is? (You are free to provide as many definitions as you have. If you provide many definitions, please explain the differences and similarities between those definitions.)
2	Basic function knowledge	Declarative	Adopted from Karahasan (2010)	List different representations of functions.	Explain the different ways in which a function can be represented.
3	Basic function knowledge	Declarative	Adapted from Karahasan (2010)	Determine if the given relations are functions and provide the justification.	Please indicate if each of the following is a function and explain your reasoning. a. An online motorcycle taxi company sets a fare of Rp 8,000 for the first 10 km and increases by Rp 2,000 for every 2 km thereafter. b. $f(x) = \begin{cases} x, \text{if } x \in \mathbb{Q} \\ 0, \text{if } \notin \in \mathbb{Q} \end{cases}$ c.
					x: 0 1 2 3 4 5 6 7 y: 0 1 2 1 0 1 2 1
4	Basic function knowledge	Declarative	Adopted from Karahasan (2010)	Define the domain and range concept.	What do you think about the domain of the function? What about the range of the function? What is the importance of them?
5	Basic function knowledge	Declarative	Adapted from Karahasan (2010)	Apply the properties of a domain of a function.	Find the function domain $f(x) = \frac{\sqrt{2+x}}{3-x}$
6	Basic function knowledge	Declarative	Adopted from Karahasan (2010)	Define the composition of functions concept.	Make a definition of composite function (symbolized $f \circ g(x)$).

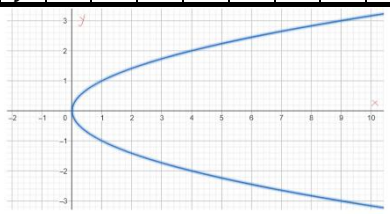
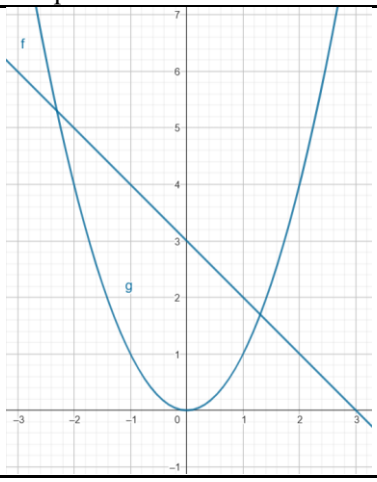
7	Operations on function	Declarative	Researchers	Apply function operations rules based on the provided function graphs.	 Considering the graphs of f and g above, determine: a. $g(2)$ b. $f+g(-1)$ c. $2f-g(1)$ d. $f \circ g(0)$ e. $g \circ f(-2)$
8	Composite function	Conditional	Adopted from Karahasan (2010)	a. Determine functions that meet the given composite function. b. Determine and describe if more than one function exists that meets the same composite functions.	Given $f \circ g(x) = \sqrt[5]{x+3}$. a. Determine f and g that meet the specified composite function. b. Can you give a different answer from part a? Explain your reasoning.
9	Properties of function	Conditional	Researchers	Apply the properties of surjective function.	Is the function $f(x) = x + 1$ from $\mathbb{N}$ to $\mathbb{N}$ a surjective function?
10	Properties of function	Procedural	Researchers	Apply the properties of injective function.	List all injective functions from $\{1.2\}$ to $\{a.b.c\}$.

For the purpose of face and construct validity, opinions by two lecturers with doctoral degree and more than ten years of experience teaching abstract algebra as validators were collected. According to their opinion, spelling mistakes were fixed; question sentences were made simpler so the students could understand them; the number of questions was decreased due to the limited work duration; and to guarantee that the developed instrument measured what it should be measuring.

In the fourth phase, one validator suggested major revision related to the question objective defining the concept of domain and range. Instead of asking definition of domain, it is better to ask

students' understanding of the concepts by determining the domain and range of the graph from the given curve. Besides, another validator also suggested revision related to surjective function concept. Instead of applying the properties of surjective function, the question could be improved by changing the objective into justify the statement about surjective function. In addition, the validator also reminded the researchers not to forget to provide information on the x and y axes in each curve image. The revised instrument to assess students' knowledge about function based on validator input is shown in the following Table 2.

Table 2: Final Questions

No	Aspect	Knowledge Type	Origin	Objective(s)	Question(s)
1	Basic function knowledge	Declarative	Adapted from Karahasan (2010)	Determine if the given relations are functions and provide the justification.	Please indicate if each of the following is a function and explain your reasoning. a. An online motorcycle taxi company sets a fare of Rp 8,000 for the first 10 km and increases by Rp 2,000 for every 2 km thereafter. b. $f(x) = \begin{cases} x, \text{if } x \in \mathbb{Q} \\ 0, \text{if } \notin \in \mathbb{Q} \end{cases}$ c. x: 0 1 2 3 4 5 6 7 y: 0 1 2 1 0 1 2 1
2	Basic function knowledge	Declarative	Adopted from Karahasan (2010)	Determine the domain and range of the graph from the given curve.	 The curve above is the representation of quadratic equation $x = y^2$. Determine the part of this curve that is the graph of a quadratic function.
3	Operations on function	Declarative	Researchers	Apply function operations rules based on the provided function graphs.	

					Considering the graphs of f and g above, determine: a. $\left(\frac{1}{2}f + g\right)(2)$ b. $f \circ g(0)$ c. $g \circ f(-2)$
4	Composite function	Conditional	Adopted from Karahasan (2010)	a. Determine functions that meet the given composite function. b. Determine and describe if more than one function exists that meets the same composite.	Given $f \circ g(x) = \sqrt[5]{x+3}$. a. Determine f and g that meet the specified composite function. b. Can you give a different answer from part a? Explain your reasoning.
5	Properties of function	Conditional	Researchers	Justify given statements about surjective functions.	Prove or give a counter-example of the statement: "Function $f(x) = x + 1$ from $\mathbb{N}$ to $\mathbb{N}$ is a surjective function."
6	Properties of function	Procedural	Researchers	Apply the properties of injective function.	List all injective functions from $\{1.2\}$ to $\{a.b.c\}$.

After the instrument was declared valid, a readability test was conducted on three students with different levels of mathematical ability: low, medium, and high levels. From the result of this test, the students did not have difficulty in understanding or interpreting each word on the instrument. So, after the readability test was conducted, the researchers did not need to revise anymore.

An instrument is crucial in assessing the quality of the study (Bastos et al., 2014). Research instrument has a very crucial role in the research process because they act as a tool to collect the necessary data. Compiling instruments into an essential stage in interrelated research procedures. This is also reinforced by previous research, that the instruments that can be used in research are instruments that have met the requirements (Desnita et al., 2021; Leite et al., 2018; Ummah & Rifai, 2021).

Conclusion and Suggestion

This research has resulted an instrument that can assess the students' MCK level about function. When developing the instrument, a number of factors were taken into consideration. The number of questions was decreased due to the limited time duration and the suggestions from the validators were implemented for better quality of questions. In addition, the instrument also passed the readability test. Thus, this instrument can be used to measure the students' level of MCK about function. In the future, this instrument could contribute to increasing the transferability of the bigger

project, the design research for facilitating students to do mathematical abstraction of the permutation group concept.

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